IV. *An Account of experiments for determining the length of the Pendulum vibrating seconds in the latitude of London. By Capt.* Henry Kater, *F. R. S.*

Read January 29, 1818.

To determine the distance between the point of suspension and centre of oscillation of a pendulum vibrating seconds in a given latitude, has long been a desideratum in science. Many experiments have been made for this purpose, but the attention of all who have hitherto engaged in the enquiry (excepting Whitehurst) appears to have been directed to the discovery of the centre of oscillation. The solution of this problem depending, however, on the uniform density and known figure of the body employed, (requisites difficult if not impossible to be ensured in practice,) it is not surprising that the experiments made by different persons should have been productive of various results.

When I had the honour of being appointed one of the committee of the Royal Society for the investigation of this interesting subject, I imagined that the least objectionable mode of proceeding would be to employ a rod drawn as a wire, in which, supposing it to be of equal density and diameter throughout, the centre of oscillation, as it is well known, would be very nearly at the distance of two-thirds of the length of the rod from the point of suspension; and I purposed by inverting the rod, and taking a mean of the results in each position, to obviate any error which might

arise from a want of uniformity in density or figure. After numerous trials however, and as frequent disappointments, I was at length convinced of the impracticability of obtaining a rod sufficiently uniform, and I was besides aware, that under certain circumstances errors might arise from this cause which it would be impossible by any method to detect.

Not feeling at all satisfied with the prospect which the use of a rod presented, I endeavoured to discover some property of the pendulum of which I might avail myself with greater probability of success; and I was so fortunate as to perceive one, which promised an unexceptionable result. It is known that the centres of suspension and oscillation are reciprocal; or in other words, that if a body be suspended by its centre of oscillation, its former point of suspension becomes the centre of oscillation, and the vibrations in both positions will be performed in equal times. Now the distance of the centre of oscillation from the point of suspension, depending on the *figure* of the body employed, if the arrangement of its particles be changed, the place of the centre of oscillation will also suffer a change. Suppose then a body to be furnished with a point of suspension, and another point on which it may vibrate, to be fixed as nearly as can be estimated in the centre of oscillation, and in a line with the point of suspension and centre of gravity. If the vibrations in each position should not be equal in equal times, they may readily be made so, by shifting a moveable weight, with which the body is to be furnished, in a line between the centres of suspension and oscillation; when the distance between the two points about which the vibrations were performed being measured, the

length of a simple pendulum, and the time of its vibration will at once be known, uninfluenced by any irregularity of density or figure.*

An unexceptionable principle being thus adopted for the construction of the pendulum, it became of considerable importance to select a mode of suspension equally free from objection. Diamond points, spheres, and the knife edge, were each considered; but as it was found difficult to procure diamond points sufficiently well executed, the knife edge was preferred, after many experiments had been made with

* In the Connoissance des Temps for 1820, is an article by M. de PRONY on a new method of regulating clocks. At the conclusion of this article is a short note, in which the author adds, "J'ai proposé en 1790 à l'Academie des Sciences un moyen "de déterminer la longueur du pendule en faisant osciller un pendule composé sur "*deux* ou trois axes attachés à ce corps. (voyez mes Leçons de Mécanique, art. "1107 et suivans) Il paroit qu'on a fait ou qu'on va faire usage de *ce moyen* en "Angleterre." On referring to the Leçons de Mécanique, as directed, I can perceive no hint whatever of the possibility of determining the length of the seconds pendulum by means of a compound pendulum vibrating on *two* axes, but it appears that the method of M. de PRONY consists in employing a compound pendulum having *three* fixed axes of suspension, the distances between which, and the time of vibration upon each, being known, the length of three simple equivalent pendulums may thence be calculated by means of formulæ given for that purpose. M. de PRONY indeed proposes employing the theorem of HUYGENS, of which I have availed myself, of the reciprocity of the axis of suspension and that of oscillation, as *one* amongst other means of simplifying his formulæ, and says, "J'ai indiqué les moyens "de concilier avec la condition à laquelle se rapportent *ces formules*, celle de rendre "l'axe moyen le reciproque de l'un des axes extrêmes; J'emploie pour les ajustemens "qu'exigent *ces diverses conditions* un poids curseur dont j'ai exposé les propriétés "dans un mémoire publié avec la Connoissance des Temps de 1817." Now it appears evident from this passage, that M. de PRONY viewed the theorem of HUYGENS *solely* with reference to the simplification of his formulæ; for had he perceived that he might thence have obtained *at once* the length of the pendulum without further calculation, the inevitable conclusion must instantly have followed that his third axis and his formulæ were wholly unnecessary.

spheres, the result of which it may not be useless for a moment to dwell upon.

It is known, that if two curved surfaces be ground together in every possible direction, they will become portions of spheres; and thus a perfect sphere may be formed by grinding a ball in a hemispherical cup. If a pendulum vibrate on such a sphere, working in a conical aperture, it is evident that the centre of the sphere will be accurately in the axis of vibration. In trying this method, however, it was found, that the friction was so considerable, as to bring the pendulum to a state of rest after a few vibrations; and when the friction was sufficiently diminished, by a contrivance which it is unnecessary to describe, the *lateral* force of the pendulum in an arc of two degrees and a half, was sufficiently powerful to carry the ball entirely out of the socket; and it was consequently evident, that though the arc of vibration might not be large enough to effect this, it must necessarily cause the ball in some degree to ascend the inclined plane of the aperture; and this consideration induced me to abandon at once a mode of suspension which I should otherwise have esteemed the best that could have been employed.

The principal objections to the use of a knife edge, appeared to be, the difficulty of forming it perfectly straight, and the possibility that it might suffer a change of figure from wear, during the experiments, which might introduce an error not to be detected. The first of these objections I found to be perfectly groundless, as a knife edge can be made so as not to deviate sensibly from a right line. The second objection would indeed be of weight, were the usual method of determining the time of vibration resorted to, by com-

paring the pendulum with a clock, at the distant intervals of 24 hours; but it will hereafter appear, that should any alteration in the form of the knife edge take place, it must become perceptible every ninth minute; in addition to which, I proposed to measure the distance of the knife edges both before and after the experiments, when any change would of course be immediately detected.

Description of the pendulum employed.

The pendulum constructed upon these principles is formed of a bar of plate brass, one inch and a half wide, and one eighth of an inch thick. Through this bar, two triangular holes are made, at the distance of 39,4 inches from each other, to admit the knife edges. Four strong knees of hammered brass of the same width as the bar, six inches long, and three quarters of an inch thick, are firmly screwed by pairs to each end of the bar, in such a manner, that when the knife edges are passed through the triangular apertures, their backs may bear steadily against the perfectly plane surfaces of the brass knees, which are formed as nearly as possible at right angles to the bar. The bar is cut of such a length, that its ends may be short of the extremities of the knee pieces about two inches.

Two slips of deal 17 inches long, and of the same thickness as the bar, are inserted in the spaces thus left between the knee pieces, and are firmly secured there by pins and screws. These slips of deal are only half the width of the bar; they are stained black, and in the extremity of each, a small whalebone point is inserted for the purpose of indicating the extent of the arc of vibration.

A cylindrical weight of brass, three inches and a half diameter, one inch and a quarter thick, and weighing about two pounds seven ounces, has a rectangular opening in the direction of its diameter, to admit the knee pieces of one end of the pendulum. This weight being passed on the pendulum, is so thoroughly secured there by means of a conical pin fitting an opening made through the weight and knee pieces, as to render any change of position impossible. A second weight of about seven ounces and a half, is made to slide on the bar near the knife edge at the opposite end; and this weight may be fixed at any distance on the bar by two screws with which it is furnished.

A third weight, or rather slider of only four ounces, is moveable along the bar, and is capable of nice adjustment by means of a screw fixed to a clamp, which clamp is included in the weight. This slider is intended to move near the centre of the bar. It has an opening, through which may be seen divisions, each equal to one twentieth of an inch, engraved on the bar; and a line is drawn on the edge of the opening to serve as an index to determine the distance of the slider from the middle of the bar.

We now come to the most important part, the knife edges. These are made of that kind of steel which is prepared in India, and known by the name of wootz. Their form is triangular, and their length one inch and three quarters. Mr. STODARD was so obliging as to forge them for me: they were made as hard as possible, and tempered by immersing them merely in boiling water.

The knife edges were ground on a plane tool, which necessarily ensured a perfectly straight edge. This was ascer-

tained by bringing the edge of the one in contact with the plane of the other, when, if no light was perceptible between them in any position, it was inferred that the edge was a right line. They were then carefully finished on a plane green hone, giving them such an inclination as to make the angle on which the vibrations are performed about 120 degrees.

Previously to the knife edges being hardened, each was tapped half way through, near the extremities, to receive two screws, which being passed through the knee pieces, drew the knife edges into close contact with them, the surfaces of both having been previously ground together to guard against any strain which might injure their figure. A plan of the pendulum is given in Plate IV. Fig. 1.

The support, and other apparatus.

The support of the pendulum is represented in Plate IV, Fig. 2. It consists of a piece of bell metal six inches long, three inches wide, and three eighths of an inch thick. An opening is made longitudinally through half the length of the piece, to admit the pendulum, and the bell metal is cast with a rectangular elevation on each side of the opening extending the whole length of the piece. Two plates of agate* were cemented to this elevated part, beds having been made to receive them, in order that their surfaces might be in the same plane with that of the bell metal. The whole was then ground perfectly flat. A frame of brass represented at Fig. 3, is attached by two opposite screws, which

* Plates of hard steel were first tried, but were found to have suffered penetration by the knife edge.

serve as centres, to the sides of the elevated part of the support, and one end of this frame being raised or depressed by means of the screw A, the pendulum when placed with its knife edge resting in Ys, at the other end of the frame, could be elevated entirely above the surface of the agate, or be gently lowered until the knife edge rested wholly upon it, and thus the knife edge was sure to bear always precisely on the same part of the agate plane, by elevating the Ys, above its surface, placing the knife edge in them, and then letting down the whole gently by means of the screw, till the Ys were completely clear of the knife edge. The support was firmly screwed to a plank which will hereafter be described.

To the kindness of HENRY BROWNE, Esq. F. R. S., I am essentially indebted for the success of the experiments which form the subject of this paper. He most obligingly allowed me the use of his house, his excellent time-pieces, and transit instrument, assisting me with indefatigable zeal by his very accurate daily observations, and intermediate comparisons for determining the rate of the clock. The house is substantially built, and is situated in a part of Portland Place not liable to much disturbance from the passing of carriages. The room in which the experiments were made is the last of two on the ground floor, communicating with each other and facing the north. The temperature consequently is very steady, and if necessary, may be raised to any given degree by a fire in the first room. The clock with which the pendulum was compared was made by ARNOLD; and in addition to the gridiron compensation for temperature, its pendulum is suspended by a spring, the strength of which is so adjusted, that the vibrations in different arcs are performed in equal times. This

clock is firmly screwed to the wall, in a recess opposite to the window. Near to this, on the wall which is at right angles to the recess, is fixed another time-piece by CUMMING, which was the property of the late General ROY, and is considered by Mr. BROWNE to be the best in his possession. Respecting this clock, it will be sufficient to remark, that three tenths of a second was the greatest variation in its daily rate from the 22d February, when the observations commenced, to the 31st July; and consequently the deviation from its mean rate during that period, did not exceed 0,15 of a second per day. This clock has been used as the standard of comparison, the time having been taken from the transit instrument by a chronometer of ARNOLD's. With such advantages it will be confessed that there can be little chance of error arising from the rate of the clock.

A plank of well seasoned mahogany, two feet wide, and three inches thick, was forcibly driven between the walls forming the sides of the recess, until it was near the top of the clock case. To this the support of the pendulum before described was firmly screwed, and carefully levelled, in such a position as to allow the pendulum to vibrate as near as possible to the clock case without touching it; and that when at rest, it might appear to an observer in front of the clock, to pass over the centre of the dial plate, its extremity reaching a little below the centre of the ball of the pendulum. Beneath, fixed to the clock case, was an arc divided into degrees and tenths, to determine the extent of the vibrations. Such a portion of the plank was cut away as was necessary to admit of the pendulum being placed on its support. A circular white disk was pasted on a piece of black paper,

which was attached to the ball of the pendulum of the clock; and this disk was of such a diameter as when both pendulums were at rest, to be just hid from an observer at the opposite side of the room, by one of the slips of deal which form the extremities of the brass pendulum.

Though there was little reason to imagine that the vibrations of the pendulum could communicate any motion to a support so firm as that which has been described, it became a point of considerable importance to verify this by actual experiment. For this purpose I had recourse to a delicate and simple instrument invented by Mr. HARDY, clock-maker, the sensibility of which is such, that had the slightest motion taken place in the support, it must have been instantly detected. This little instrument is represented in Plate IV. Fig. 4. It consists of a steel wire, the lower part of which inserted in the piece of brass which serves as its support, is flattened so as to form a delicate spring. On the wire, a small weight slides, by means of which it may be made to vibrate in the same time as the pendulum to which it is to be applied as a test. When thus adjusted, it is placed on the material to which the pendulum is attached; and should this not be perfectly firm, its motion will be communicated to the wire, which in a little time will accompany the pendulum in its vibrations. This ingenious contrivance appeared fully adequate to the purpose for which it was employed, and afforded a satisfactory proof of the stability of the point of suspension.

A firm triangular wooden stand, as high as the ball of the pendulum, was screwed to the floor at the distance of nine feet in front of the clock. This served as a support, to which was attached a small telescope, magnifying about four times,

which was capable of a horizontal motion on its axis, a vertical motion, and a motion at right angles to the line of sight. In the focus of the eye-glass was a diaphragm forming a perpendicular opening, the sides of which were parallel, and capable of being placed nearer, or further asunder. The edges of this diaphragm were adjusted so as to form tangents to the horizontal diameter of the white disk, and consequently to coincide with the edges of the slip of deal. When, therefore, both pendulums were at rest, nothing was visible through the telescope, excepting the divided arc for ascertaining the extent of the vibrations, and which was seen through a horizontal opening made for that purpose in the top of the diaphragm.

Method of determining the number of vibrations made by the pendulum in 24 hours.

If both pendulums be now set in motion, the brass pendulum a little preceding that of the clock, the following appearances may be remarked. The slip of deal will first pass through the field of view of the telescope at each vibration, and will be followed by the white disk. But the distance between the centres of suspension and oscillation in the brass pendulum being rather the longer, the pendulum of the clock will gain upon it, the white disk will gradually approach the slip of deal, and at length, at a certain vibration, will be wholly concealed by it. The minute and second at which this total disappearance is observed, must be noted. The pendulums will now be seen to separate, and after a time will again approach each other, when the same phenomenon will take place. The *interval* between the two coincidences in seconds, will give the number of vibrations made by the pendulum of the clock; and the number of oscillations of the

brass pendulum, in the same interval, may be known by considering that it must have made two oscillations less than the pendulum of the clock. Hence by simple proportion, as the vibrations made by the pendulum of the clock, are to the number of vibrations made by the brass pendulum, so are the vibrations made by the pendulum of the clock in 24 hours, to those of the brass pendulum in the same period.*

Many experiments were made in order to select such a distance of the knife edges as might give an interval which would allow of the determination of the time of coincidence without an error of a single second,† and yet afford a convenient number of intervals before it should become necessary to renew the motion of the pendulum. At the first coincidence, the velocity of the brass pendulum, at the lowest part of the arc, must not exceed that of the pendulum of the clock, otherwise the disk would disappear for an imperceptible time, and then re-appear; and this limits the extent of the arc of vibration.

Again; the observations must not be continued beyond a certain diminution of the arc of vibration, otherwise the *space*, which the pendulum of the clock has to gain on the brass pendulum in one vibration, becomes so small as to render the observation of the time of coincidence in some degree uncertain; and, should the *space* be so far diminished as to be less than the error or deviation from a right line, which would

* In order to render the calculation more easy, the clock has always been supposed to keep mean time, or to make 86400 vibrations in 24 hours, and the variation from this number, or the rate of the clock (being a very small quantity) has been afterwards applied as a correction.

† The *principle* on which this method of coincidences is founded, was employed by Dr. WOLLASTON, in May 1808, in some experiments in which he was then engaged, the moment of coincidence being determined however by sound instead of sight.

probably take place in the adjustment of the sides of the diaphragm, the end of the pendulum, and the disk, the results would be erroneous, as the interval would go on increasing till the pendulum came to a state of rest.

The interval which best fulfilled these conditions was found to be about 530 seconds. This admitted five coincidences (affording four intervals) to be taken before the arc became too small for the observations to be continued with safety. With this interval an error of one second in the time of coincidence would occasion an error of only 0,63 in the number of vibrations in 24 hours.

Here it must be evident that no sensible alteration could take place in the knife edge during the experiments without its becoming perceptible at every coincidence, since the number of vibrations in 24 hours deduced from each interval, must vary with any change in the form of the knife edge.

The following was the method pursued in making the observations. The small weight or slider being placed with its index at a certain distance (say one inch and a half) from the middle of the pendulum towards the great weight, and the second weight about five inches from the knife edge, the Ys of the support were elevated, the knife edge of the pendulum was placed in them, with the great weight *above*, and the frame gently lowered till the knife edge was left on the surface of the agate. The requisite adjustments of the telescope having been made, the pendulum was set in motion in an arc not exceeding one degree and four tenths, in order that its velocity might not be greater than that of the pendulum of the clock.

The minute and second, at which the disk ceased to be visible, was then carefully noted; and the arc of vibration seen through the telescope, the height of the barometer, and the temperature indicated by a thermometer suspended on the clock case near the middle of the brass pendulum, were also observed and registered. Five successive coincidences were thus taken, and the number of vibrations in 24 hours was deduced from them in the manner before described; but the vibrations thus obtained being made in different arcs, it became necessary to apply a correction to determine what they would have been in an arc infinitely small. For this correction I might have used a formula depending on the decrease of the arcs in geometrical progression, whilst the times decrease in arithmetical; but as there is an uncertainty in observing the arc of vibration amounting to one or two hundredths of a degree, this method, though more perfect in theory, would have been an unnecessary refinement in practice.

The error arising from the greater length of the vibration in a circular arc, being nearly as the square of the arc, if the mean of the observed arcs at the commencement and end of each interval be taken, and its square multiplied by 1,635 (the difference between the number of vibrations made by the pendulum in 24 hours, in a cycloid and in an arc of one degree,) the required correction will be obtained, to be added to the number of vibrations before computed.

The mean of these last results being taken, and also the mean of the observed temperatures at the first and last coincidences, the number of vibrations in 24 hours was obtained

at a certain temperature, and altitude of the barometer, in an infinitely small arc, the great weight being *above*.

The frame of the support was now elevated, the pendulum was inverted, placed in the Ys, with the great weight *below*, and the knife edges being gently let down as before on the agate plane, the same process with respect to the observations was followed, which has just been described. And if the mean temperature differed from that in the former position of the pendulum, the mean number of vibrations was corrected for such difference of temperature, the expansion of the pendulum being known by experiments hereafter to be detailed, and consequently the gain or loss in 24 hours by a given change of temperature.

The mean number of vibrations thus found, differing from that given in the former position of the pendulum, the second weight was moved, the number of vibrations again determined, and the pendulum being inverted, the process was repeated until the vibrations in 24 hours, in either position of the pendulum were brought as near to an equality as could readily be effected by means of this weight; it was then firmly secured in its place.

Whatever alteration may be made in the arrangement of the weights, the effect on the vibrations (except in one particular instance) will be the same in both positions of the pendulum, always increasing or diminishiug their number in both cases, though in different degrees; and, the vibrations will be least affected by such change when the great weight is below, and will consequently be nearest to the truth in this position. No doubt, therefore, can arise, as to the kind of correction required. The number of

vibrations after the adjustment by the second weight has been completed, must be left *in defect*, for a reason which will be immediately apparent.

There is a point in the pendulum where the effect of the slider in increasing the number of vibrations is a maximum; and it appears from Dr. YOUNG'S investigations, that this point in one position of the pendulum is different from that in the other. *Very near* either of these points, the pendulum being in its corresponding position, the motion of the slider produces scarcely any change in the number of vibrations; but the slider being then more distant from the point of maximum belonging to the other position of the pendulum, the corresponding increase of the number of vibrations arising from such motion of the slider, will, in that position be very perceptible.

In the present instance, the point of maximum in either position of the pendulum, is about four tenths of an inch below the middle, and consequently the distance of the two points from each other, is about eight tenths of an inch. The slider which had remained stationary during the adjustment of the second weight at about one inch and a half from the middle of the pendulum towards the great weight, must now be shifted, (say one inch) towards the middle of the pendulum, in order to increase the number of vibrations which it may be recollected were left in *defect*, so that they may be in *excess*. It is evident that the true number of vibrations will be found, when the slider is somewhere between its first and second position. Let the slider be now placed half-way between these two points. If the number of vibrations in this

third position be still in excess, the truth will lie between the first and third positions of the slider. And thus by continually bisecting with the slider, the distance of the two last found points, the number of vibrations when the great weight is *below*, will rapidly approach the truth, being alternately in defect and in excess; and when the approximation is such as that the difference in either position of the pendulum becomes inconsiderable, the vibrations, when the great weight is below, may be taken for the truth; and thus the number of vibrations in 24 hours, of a pendulum equal in length to the distance between the knife edges, will be known at a certain temperature, and at an observed height of the barometer. The general arrangement of the apparatus is represented by Plate V.

Of the apparatus and methods employed for the measurement of the distance between the knife edges, and for the comparison of the British standard measures of the highest authority.

The microscopes used for this purpose, were made by Mr. THOMAS JONES, of Cockspur-street. They are both furnished with cross threads of spider's web, as well as with a single thread for the purpose of bisecting a dot if required, and are in other respects of a similar construction with those described by Sir GEORGE SHUCKBURGH EVELYN, in the Philosophical Transactions for 1798, but are more powerful, and the micrometer is capable of far greater precision.

The object glass of the micrometer microscope is of one inch focus, the distance from the object glass to the spider's threads 3,25 inches, the focus of the compound eye-glass rather less than one inch, the magnifying power 18 times.

In the other microscope, which I shall call the fixed microscope, the object glass is of three quarters of an inch focus, and the magnifying power consequently greater. The micrometer head is divided into one hundred parts.

Each microscope slides in a tube, which is fixed in a plate of brass forming part of its support; and this plate moves in a dovetail, by which the microscope may be brought over the object to be viewed, when it is firmly clamped by a screw.

A piece of well seasoned mahogany, four inches and three quarters, by three inches, served as a beam to which the supports of the microscopes were screwed, their centres being 39,4 inches asunder.

Two screws with milled heads, supported the extremities of the beam in front, and a piece projecting from the middle of the beam behind, served as a third leg. By means of the screws, the focus of either microscope could be nicely adjusted at pleasure, without any risk of altering their distance from each other.

My first object was to ascertain the degree of precision of which vision is capable when assisted by the microscope. For this purpose, a very fine line was drawn on a polished piece of brass, and the microscope being carefully adjusted so as to be free from parallax, by causing the image of the line to bisect the angles formed by the spider's threads, moving the eye to the right and left and remarking whether the image changed its situation, and if it did, varying the distance of the microscope from the object accordingly, until the line appeared stationary, the micrometer screw was turned back, and the spider's threads brought up again till the angle

formed by them appeared to be accurately bisected by the line. The division of the micrometer was then noted, and this was repeated several times with scarcely a sensible difference in the result; and thus I assured myself that no error worthy of remark was to be apprehended from imperfection of vision.

The next step was to determine the value of one division of the micrometer head. By the kind interest of Sir JOSEPH BANKS, I was favoured with the use of the standard scale which belonged to the late Sir GEORGE SHUCKBURGH EVELYN, and which is described in the Philosophical Transactions for 1798. This scale, the work of Mr. TROUGHTON, is second to none in the kingdom in point of accuracy of division, and is too well known to render any further remark necessary. The microscope being carefully adjusted for parallax, one inch, from the 39th to the 40th, was measured by successive tenths, and the mean taken as the value of one tenth of an inch. The measurement of the same inch was repeated ten times at different periods, the microscopes being previously adjusted anew each time for parallax. The mean results of such measurements are as follows.

Divisions of the micrometer to $\frac{1}{10}$ of an inch.

	2335,00
	2333,75
	2337,55
	2337,32
	2334,50
	2336,90
	2335,75
	2338,30
	2335,85
	2337,85
Mean	2336,277

Hence, the value of one division of the micrometer appears to be $\frac{1}{23363}$ of an inch.

In the course of these measurements, differences occurred for which I was at a loss to account; but at length it appeared that they were to be attributed to remaining parallax; for whatever care be taken in adjusting the microscope, it is scarcely possible to bring the image of the object precisely in the same plane with the threads, and the image will consequently be of various dimensions, according to its distance from this plane. Unless, therefore, the most minute attention be paid to the adjustment for parallax, the error arising from this cause will be considerable; and I may here remark that I believe the difficulty of bringing the image into the plane of the threads, to be the source of by far the most serious errors to which measurements by means of microscopes are liable.

I had now to examine the equality of the threads of the micrometer screw. For this purpose, two fine lines were drawn near each other on a piece of brass, and the micrometer being turned back as far as it would go, the distance of the lines was carefully measured; and this was repeated, proceeding through the whole length of the screw, always advancing the micrometer one revolution previous to each successive measurement. The result of this severe test will best appear by giving the numbers themselves.

Divisions of the micrometer.

502,0
501,5
501,0
502,0
501,5
502,0
502,0
502,5
502,0
502,0
502,5
501,0
501,5
502,0
501,0
502,5
501,0
500,0
500,0
500,5

Mean 501,5

The mean is 501,5, and the greatest difference from the mean only one division, amounting to $\frac{1}{23363}$ of an inch, a degree of accuracy truly surprising, when it is considered that all errors of observation are included in this minute quantity.

Comparison of the different standards.

The microscopes being placed at the distance of 39,4 inches, were advanced by single tenths, from zero of the scale through the space of two inches, and the mean of twenty measurements thus obtained being compared with the distance from zero to 39,4 inches, this last was found to be

in defect 1,2 divisions of the micrometer, or ,00005 of an inch. And as this is the portion of the scale employed in ascertaining the distance between the knife edges, this difference must ultimately be subtracted to obtain the distance of the knife edges, in parts of the mean value of the scale.*

From the high importance which attaches to General ROY'S scale, as having formed the basis of the Trigonometrical Survey of the kingdom, I was particularly desirous of comparing it with that of Sir G. SHUCKBURGH, in order that I might be enabled to give the length of the pendulum in parts of that standard which constitutes the foundation of one of the most important scientific operations ever carried on in this country. Fortunately, this scale was purchased at the sale of General ROY'S effects by Mr. BROWNE, who readily confided it to my care. From the mean of a number of comparisons, I found the distance from zero to 39,4 of General ROY'S scale, equal to 39,40144 of Sir G. SHUCKBURGH'S standard.†

* From an examination of this scale by the late Sir G. SHUCKBURGH, it appears that the greatest liability to error is ,00033 of an inch, or as corrected by Mr. TROUGHTON, ,000165 of an inch, the chances against which are as 9 to 1.

† The very great difference between this result and that stated by Sir GEORGE SHUCKBURGH, in the Philosophical Transactions for 1798, renders it necessary for me briefly to detail the manner in which the comparisons were made. The two scales were placed in contact, and remained thus for twenty-four hours; after which, sixteen comparisons were taken in the course of the day; but these were rejected in consequence of the temperature having increased six degrees during the operation. When the scales had been together forty-eight hours, sixteen other comparisons were made during two succeeding days, the thermometer remaining steadily at 70°. The greatest difference between any one of these last and the mean result, did not amount to four divisions of the micrometer. The mean of the first set of observations ex-

The standard yard made by BIRD in 1758, for the House of Commons, better known by the name of BIRD's Parliamentary standard, is little adapted for measurements where great precision is necessary. The yard is determined by two large dots made on gold pins, which are let into a bar of brass. The mean of a number of bisections of these dots gave their distance equal to 36,00016 inches of Sir GEORGE SHUCKBURGH's scale.

Measurement of the pendulum.

The pendulum was let into a solid piece of mahogany edgewise, to such a depth that the knife edges were about one twentieth of an inch above its surface. To one end of the pendulum, a common spring steelyard was attached by its hook, and a string being passed through the ring, and fastened to an upright piece of wood screwed to the end of the mahogany case, the pendulum was extended by a force rather greater than its own weight (about ten pounds), and consequently, no error (if any such were to be apprehended)

ceeded that of the last by ,00017 of an inch. Imagining that the difference between Sir GEORGE SHUCKBURGH's result and mine, might possibly be occasioned by an error in the divisions bounding that part of General ROY's scale which I had employed, I compared it with various other portions, and found no greater difference than might have been expected from unavoidable imperfection of division. It is to be presumed then, that the error into which Sir GEORGE SHUCKBURGH appears to have fallen, must have arisen from the two scales not having been of the same temperature at the time they were compared, particularly as Sir GEORGE SHUCKBURGH's is by far the most massive of the two. I may here add, that last winter wishing to know whether the expansion of the two scales was equal, I roughly compared them together once, at the temperature of 33°, when it appeared that 42 inches on General ROY's scale, was equal to about 42,001 inches of Sir GEORGE SHUCKBURGH's standard.

could arise from a difference in the length of the pendulum in its vertical and horizontal positions.

The knife edges were fixed as nearly as could be done by mechanical means, at right angles to the bar of the pendulum; but the bar being flexible, they would most probably, when the pendulum was extended for the purpose of measurement, be found to be not precisely parallel to each other, and would consequently require some adjustment. To effect this, two opposite screws were passed through the sides of the mahogany case, so as to act in a transverse direction against that extremity of the pendulum which was next the steelyard, and the microscopes being brought over the extreme points of the knife edges, alternately on either side of the bar, the requisite parallelism was readily obtained by means of the screws, sufficient room having been left in the mahogany case for the very small motion of the extremity of the pendulum which might be found necessary. This arrangement is represented in Plate IV. Fig. 5.

To obtain the distance between the knife edges, two different methods were used. For the first, four rectangular pieces of brass were prepared, about half an inch square. Very near to the perfectly straight edge of each, a fine line was drawn, to be viewed through the microscope, and these lines were each crossed at right angles by two others, intended to indicate that part of the first line from which the measurements were to be taken. These pieces were marked A, *a*, and B, *b*.

The pieces A and *a*, being placed with their edges in contact, in which position they were kept by the pressure of a spring, the distance between the fine lines first drawn was

carefully measured with the micrometer, and from a mean of eight observations, the greatest difference between which did not exceed one division, was found to be 329,09 divisions.

The same was done with the pieces B and b, and the distance of the lines from a mean of sixteen observations appeared to be 366,96 divisions.

The knife edges being adjusted as nearly as possible parallel to each other, the pieces A, a, and B, b, were placed in contact with those parts of the knife edges on either side of the bar, on which the vibrations were to be performed, and were retained in their places by the pressure of slight springs, attached to the mahogany case.

The microscopes were now brought over the pieces A and a, so as for the lines before described to bisect the cross threads, when the division of the micrometer was noted.

The same was done with the pieces B and b; and the division of the micrometer was also registered.

The pendulum being removed, the standard scale* was placed beneath the micrometer, and its zero being made to bisect the angles of the fixed microscope, the cross threads of the micrometer microscope were brought to 39,4 of the scale, and the revolutions and parts of the micrometer were noted.

From these data, and the respective distances of the lines on A and a, and on B and b, when the pieces were in contact, the distance of the knife edges on either side of the bar may be readily obtained, and the mean being taken, will obviously correct any error arising from a want of perfect parallelism in the knife edges.

It is very generally believed that measurements from a

* The scale constantly referred to, is Sir George Shuckburgh's standard.

knife edge, or from a line terminating a surface, are liable to much uncertainty from what has been called *irradiation*, or indistinctness of the image. But this is by no means the fact; for if the reflection of light from the knife edge be prevented, and it be viewed on a white ground, it may be made to bisect the cross threads of the microscope, with nearly the same precision as could be attained by the use of a line. There is, however, a correction necessary to be applied in this case, and I shall proceed to describe the method employed for ascertaining its amount.

A slip of writing-paper was pasted on the mahogany case, under each knife edge, extending beyond it about the tenth of an inch, and adjoining, was a piece of black paper to prevent the reflection of light on the knife edge from the surrounding objects. The knife edge now appeared through the microscope, as a well defined dark object on a white ground.

Marks were made on the paper close to the knife edges at equal distances on each side of the bar. These were intended to indicate those parts of the knife edges equally distant from the middle, from which the measurements were to be taken.

The knife edges being adjusted parallel to each other, in the manner before described, the microscopes were brought successively over such marks on the paper, as were at the same distance from the bar, and the mean of each pair of observations being referred to the scale, gave a distance of the knife edges free from any error which would be occasioned by a want of parallelism.

The knife edges bisecting the cross threads of the micro-

scopes, pieces of black paper were slid beneath them, when they appeared to start forwards towards each other, the images continuing perfectly sharp and well defined.

The distance between the knife edges appeared to be now considerably less than before; and it remained to determine the difference, in order to apply its half, as a correction to the distance first obtained.

For this purpose, the reading of the micrometer was taken when the knife edges were viewed as dark objects on a white ground, and also when they were seen as light objects on a black ground. The difference of such readings will obviously give double the correction required. The results are contained in the following table.

Divisions of the microm. the ground being *white*.	Divisions of the microm. the ground being *black*.	Difference.
32,0	44,0	12,00
19,5	30,0	10,50
17,5	28,0	10,50
16,5	27,7	11,20
12,5	25,0	12,50
12,5	22,0	9,50
12,0	23,0	11,00
10,0	21,0	11,00
9,7	18,0	8,30
5,5	19,0	13,50
5,7	16,5	10,80
5,0	16,5	11,50
	Mean	11,03

From the above table it seems that 5,51 divisions (or ,000236 of an inch) are to be subtracted from the distance

obtained when the knife edges are viewed as dark objects on a light ground; and on the contrary, the same quantity to be added when they are seen as light objects on a dark ground.

From the few experiments I have made, this quantity appears to be the same, whatever may be the relative illumination of the object and its ground, so long as the difference of character is preserved. On the cause of this extraordinary fact I can hazard no conjecture, and it remains an interesting subject for future investigation.

Of the expansion of the pendulum.

The composition of brass is so various, that probably no two specimens possess precisely the same rate of expansion. It became therefore necessary to determine the expansion of the pendulum by direct experiment, instead of adopting the conclusions of others, and for this purpose the following method was used. A trough of deal was made of a length sufficient to receive the bar intended for the pendulum, which was placed edgewise in the middle of the trough, being secured at one end by wedges on both sides. The bar was supported on small pieces of glass tube, serving as rollers to prevent friction, and the trough was of the same depth as the width of the bar.

Two transverse lines were drawn near the extremities of the edge of the bar, distant from each other 49,5 inches, and a third line was subsequently drawn one inch beyond. The microscopes were placed over the lines, and left, together with a thermometer, for twenty four hours previous to the experiment.

The temperature being then registered, and the micro-

scopes having been examined to see that the lines bisected the angles formed by the spider's threads, the trough was filled with hot water to the edge of the bar, and two thermometers were placed in it, one just beneath the surface of the water, and the other at the bottom of the trough. The bar rapidly expanded, and the line on it was followed by the micrometer till it became stationary. The bisection was then perfected, and the mean of the degrees shewn by the thermometers registered, together with the number of revolutions and parts made by the micrometer. The whole was now suffered to remain till the temperature had become several degrees lower, when the contraction of the bar, occasioned by such decrease of temperature was measured, and thus several successive observations were made, which are contained in the following table.

Distance between the lines on the bar, 49,5 inches.				
Highest Temp.	Lowest Temp.	Diff of Temp.	Divis. of the micrometer.	Expansion in parts of the length for each degree.
° 96	° 43	° 53	620	,000010116
93	43	50	580	,000010030
Distance between the lines on the bar, 50,5 inches.				
91	43	48	600	,000010616
89	84	5	70	,000011890
83	75	8	89	,000009448
75	61	14	149	,000009038
80	44	36	400	,000009436
80	60	20	215	,000009129
73	60	13	152	,000009930
			Mean of the whole,	,000009959

The mean, ,000009959 may be taken as the expansion of the pendulum in parts of its length due to a change of temperature of one degree of the thermometer.

Of the method of deducing the length of the pendulum vibrating seconds.

The distance between the knife edges was taken when the standard scale and the pendulum were both of the same temperature; and as this temperature did not differ considerably from 62°, the difference in the rate of the expansion (if any) between the pendulum and the scale may be neglected as perfectly insensible, and 62° be considered as the temperature of measurement.

The number of vibrations made by the pendulum in 24 hours, having been determined at a different temperature, the length of the pendulum will be greater or less as the temperature of observation exceeds or falls short of 62°; and by applying the expansion due to such difference of temperature, derived from the experiments contained in the preceding article, the distance of the knife edges, or length of the pendulum will be known for the temperature at which the number of vibrations was determined, whence the length of the pendulum vibrating seconds may be readily deduced, the lengths of pendulums being to each other inversely in the duplicate ratio of the number of their vibrations in 24 hours.

Of the correction for the buoyancy of the atmosphere.

The length of the pendulum thus found, differing from what it would have been had the vibrations been made in

vacuo, it is necessary to apply to it a correction for the buoyancy of the atmosphere.

For this correction, the weight of the pendulum, compared with that of air, at the time of observation, must be known.

The pendulum being composed of different kinds of brass, the specific gravity of each part was carefully determined, and from thence the specific gravity of the whole mass.

Part of the pendulum.	Weight in air.	Specific gravity.
	lb.	
3 weights (cast brass)	3,14	8,417
4 knee pieces (cast brass)	3,13	7,816
Bar (plate brass)	3,30	8,532

From the above data, the specific gravity of the pendulum is 8,469; or the weight of the pendulum compared with water is as 8,469 to 1.

It has been determined by Sir GEORGE SHUCKBURGH (Phil. Trans. for 1777) that water is 836 times heavier than air, when the thermometer is at 53°, and the barometer at 29,27 inches. But the specific gravity of air varies directly as the height of the barometer, and inversely as its expansion, which is known to be $\frac{1}{480}$th part of its bulk for each degree of FAHRENHEIT; consequently, for any other state of the barometer and thermometer, the number 836, will vary *inversely* as the height of the barometer, and *directly* $\frac{1}{480}$th part for each degree of the thermometer above 53°.

Thus the specific gravity of water, compared with that of air, may be known for the temperature and altitude of the barometer at the time of observation; and multiplying this

by the specific gravity of the pendulum, the ratio of the weight of the pendulum compared with that of air will be obtained.

This ratio will express the diminution of the force of gravity arising from the buoyancy of the atmosphere; and in order that the number of vibrations may be the same in vacuo as in air, the length of the pendulum must be increased in the proportion of this ratio to 1, the lengths of pendulums vibrating in the same time, varying directly as the force of gravity.

Detail of the experiments.

In the first experiments which were made with the pendulum, it has been already observed that the knife edges rested on plates of hard steel, but as these at the conclusion were found to have suffered penetration in no slight degree, planes of agate were substituted for them, and the results having thus been rendered doubtful, were deemed inadmissible. It may not however be irrelevant to remark, that the distances of the knife edges obtained by the two methods which have been before described, did not differ quite one ten thousandth of an inch; and, that on re-measurement after the knife edges had been used a very considerable time, their distance was found to be increased by wear, four divisions only of the micrometer, or not quite two ten thousandths of an inch. The length of the seconds pendulum deduced from these first experiments, differed from the result of the observations about to be detailed, only two ten thousandths of an inch *in defect*. I nevertheless think it useless to insert these first experiments, as the near approximation of the result cannot but be deemed to have been in some degree accidental.

In repairing the knife edges after the termination of the first series of experiments, one of them was broken, and when it was replaced by another, the distance between them was increased about one hundredth of an inch, a circumstance which proved rather gratifying than otherwise, as it afforded a pendulum differing in length from the former one, and which yet gave nearly the same result.

June 9th, 1817, the knife edges being adjusted parallel to each other, and the scale and pendulum having remained together for several preceding days, the pieces A, a, and B, b, were applied to the knife edges in the manner described in the former part of this paper, and the following measurements were taken.

Distance from A to a, 329,06 divisions.
B to b, 366,97

Date.	Readings of the micrometer.			Divisions + 39,4 in.
	A to a.	B to b.	Scale.	
June 9th.	27,0	62,0	653,0	956,51
	21,0	52,0	642,7	954,21
	13,0	52,0	642,5	958,01
	12,0	48,0	638,5	956,51
	18,0	50,0	643,0	957,01
	18,0	50,0	642,0	956,01
The pieces changed.				
10th.	65,5	112,7	698,0	956,91
	64,0	112,5	696,0	955,76
	61,0	106,2	693,2	957,61
	64,5	108,0	693,5	955,26
	65,0	106,2	694,5	956,91
	67,2	107,0	696,0	956,91
Mean of the whole				956,47

Hence the distance between the knife edges is 39,4 inches +956,47 divisions of the micrometer.

June 12th, the knife edges having been adjusted parallel to each other, the following measurements were taken, the knife edges being viewed as *dark* objects on a *white* ground.

Dark on a white ground.					
Date.	Readings of the micrometer.				Divisions +39,4 in.
	Near side of the bar.	Further side of the bar.	Mean.	Scale.	
June 12	50,0	50,0	50,00	1006,5	956,50
	50,0	50,0	50,00	—	956,50
	50,0	50,0	50,00	—	956,50
	49,0	50,0	49,50	—	956,50
12	46,5	44,0	42,25	1001,0	955,75
	44,5	44,5	45,50	1001,0	955,50
	42,0	43,0	42,50	1003,0	960,50
	43,0	43,0	43,00	1003,0	960,00
13	37,5	38,0	37,75	994,0	956,25
	35,0	39,0	37,00	993,5	956,50
	38,0	35,0	36,50	1001,0	964,50
	38,0	38,0	38,00	—	963,00
14	25,5	27,5	26,50	987,0	960,50
	25,0	26,5	25,75	987,5	961,75
	24,0	25,2	24,60	—	962,90
	25,0	25,7	25,25	—	962,25
14	79,5	78,0	78,75	1042,0	963,25
	76,0	75,0	75,50	—	966,50
	72,0	71,5	71,75	1035,2	963,45
	74,0	73,5	73,75	—	961,45
				Mean of the whole	960,00
				Correction for irradiation (see page 59)	—5,51
					954,49

By the foregoing measurements, the distance between the knife edges appears to be 39,4 inches +954,49 divisions.

The pendulum was now placed on its support, and the following experiments made for equalising the number of vibrations.

Slider 18 divisions. Clock losing 0″,33 on mean time.				Great weight *above*.					Barometer 29,7.
	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
June 19th.	°	m. s.	°	°				s	
	66,8	29.12	1,29	1,18	518	516	86066,40	2,28	86068,68
		37.50	1,07	1,00	520	518	86067,70	1,63	86069,33
	66,8	46.30	0,93						
								Mean	86069,00
								Clock	— 0,33
	66,8	mean							86068,67
				Great weight *below*.					
	66,6	5,59	1,25	1,19	513	511	86063,16	2,32	86065,48
		14,32	1,13	1,06	512	510	86062,50	1,84	86064,34
	66,7	23,4	1,01						
								Mean	86064,91
								Clock	— 0,33
								Temp.	— 0,04
	66,7	mean							86064,54

Here the vibrations were in excess; the slider was therefore placed at 29 divisions, and the second weight moved nearer to its knife edge.

Slider 29 divisions. Clock losing 0″,33 on mean time.

Great weight *above*. Second weight moved.

Barometer 29,7.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
June 19th	° 67,0	m. s. 57.42	° 1,23					s	
		6. 8	1,04	° 1,13	506	504	86058,49	2,08	86060,57
	66,9	14.37	0,88	0,96	509	507	86060,50	1,51	86062,01
								Mean	86061,29
								Clock	— 0,33
	66,9	mean.							86060,96

Great weight *below*.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
	67,1	24.39	1,14						
		33. 5	1,01	1,07	506	504	86058,49	1,87	86060,36
	67,1	41.32	0,93	0,97	507	505	86059,16	1,51	86060,67
								Mean	86060,51
								Clock	—0,33
								Temp.	+0,09
	67,1	mean.							86060,27

The number of vibrations being still in excess, the second weight was moved again, the slider remaining as before.

Slider 29 divisions. Clock losing 0″.33 on mean time.		Great weight *above.* Second weight moved.							Barometer 29,7.
	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
June 19th.	° 67,3	m. s. 51. 7	° 1,21	°				s.	
		59 30	1,02	1,11	503	501	86056,47	2,02	86058,49
		7.54	0,88	0,95	504	502	86057,16	1,48	86058,64
		16.20	0,73	0,80	506	504	86058,49	1,05	86059,54
	67,3	24.47	0,63	0,68	507	505	86059,16	0,75	86059,91
								Mean	86059,14
								Clock	— 0,33
	67,3	mean							86058,81
		Great weight *below.*							
	67,4	30. 8	1,14						
		38.32	1,05	1,09	504	502	86057,16	1,94	86059,10
		46.56	0,94	0,99	504	502	86057,16	1,60	86058,76
		55.22	0,84	0,89	506	504	86058,49	1,30	86059,79
	67,4	3.48	0,75	0,79	506	504	86058,49	1,02	86059,51
								Mean	86059,29
								Clock	— 0,33
								Temp.	+ 0,04
	67,4	mean							86059,00

Slider 29 divisions. Clock losing 0″,26 on mean time.	Great weight *above*.								Barometer 29,76
	Temp.	Time of coincidence.	Arc of vibration	M arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
June 20th.	° 68,7	m. s. 1.17	° 1,22	°				s.	
		9.40	1,04	1,13	503	501	86056,47	2,08	86058,55
		18. 3	0,88	0,96	503	501	86056,47	1,51	86057,98
		26.28	0,73	0,80	505	503	86057,82	1,05	86058,87
	68,7	34.52	0,63	0,68	504	502	86057,16	0,75	86057,91
								Mean	86058,33
								Clock	— 0,26
	68,7	mean							86058,07
	Great weight *below*.								
	68,4	22.4	1,19						
		30.28	1,07	1,13	504	502	86057,16	2,08	86059,24
		38.52	0,94	1,00	504	502	86057,16	1,63	86058,79
		47.16	0,86	0,90	504	502	86057,16	1,38	86058,54
	68,5	55.41	0,78	0,82	505	503	86057,82	1,10	86058,92
								Mean	86058,87
								Clock	— 0,26
								Temp.	— 0,09
	68,5	mean							86058,52

The number of vibrations being now in both positions sufficiently near each other, and in *defect*, the second weight was secured in its place.

Slider 23 divisions. Clock losing 0″,30 on mean time.			Great weight *above*.						Barometer. 29,76.
	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
	°	m. s.	°	°				s.	
June 20th.	68,7	48.25	1,23	1,13	504	502	86057,16	2,08	86059,24
		56.49	1,03	0,97	506	504	86058,49	1,54	86060,03
		5.15	0,91	0,82	507	505	86059,16	1,10	86060,26
		13.42	0,73	0,68	506	504	86058,49	0,75	86059,24
	68,7	22 08	0,64						
								Mean	86059,69
								Clock	— 0,30
	68,7	mean							86059,39
A			Great weight *below*.						
	68,8	31 17	1,22	1,16	504	502	86057,16	2,20	86059,36
		39 41	1,10	1,04	505	503	86057,82	1,76	86059,58
		48.6	0,99	0,94	506	504	86058,49	1,44	86059,93
		56.32	0,90	0,84	506	504	86058,49	1,15	86059,64
	68,9	4.58	0,79						
								Mean	86059,63
								Clock	— 0,30
								Temp.	+ 0,09
	68,9	mean							86059,42

Slider 23 divisions. Clock losing 0″,20 on mean time.			Great weight *above*.					Barometer 29,86.	
	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
	°	m. s.	°	°				s	
June 21st.	71,2	54. 8	1,23	1,13	503	501	86056,47	2,08	86058,55
		2.31	1,03	0,96	502	500	86055,78	1,15	86056,93
		10.53	0,89	0,81	504	502	86057,16	1,07	86058,23
		19.17	0,73	0,68	504	502	86057,16	0,75	86057,91
	71,3	27.41	0,63						
								Mean	86057,90
								Clock	— 0,20
	71,3	mean							86057,70
B			Great weight *below*.						
	71,0	12.52	1,20	1,14	503	501	86056,47	2,12	86058,59
		21.15	1,08	1,02	503	501	86056,47	1,70	86058,17
		29.38	0,97	0,92	503	501	86056,47	1,38	86057,85
		31.1	0,88	0,83	504	502	86057,16	1,13	86058,29
	71,1	46.25	0,79						
								Mean	86058,22
								Clock	— 0,20
								Temp.	— 0,09
	71,1	mean							86057,93

Slider 23 divisions. Clock losing 0",20 on mean time

Great weight *above*.

Barometer 29,86.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
June 21st.	° 71,4	m. s. 36.6	° 1,22					s.	
		44.28	1,03	1,12	502	500	86055,78	2,05	86057,83
		52.51	0,87	0,95	503	501	86056,47	1,48	86057,95
		1.14	0,73	0,80	503	501	86056,47	1,04	86057,51
	71,4	9.40	0,62	0,67	506	504	86058,49	0,73	86059,22
								Mean	86058,13
								Clock	— 0,20
	71,4	mean							86057,93
C.		Great weight *below*.							
	71,5	20.24	1,19						
		28.46	1,07	1,13	502	500	86055,78	2,08	86057,86
		37. 9	0,94	1,00	503	501	86056,47	1,63	86058,10
		45.32	0,85	0,89	503	501	86056,47	1,29	86057,76
	71,6	53.55	0,78	0,81	503	501	86056,47	1,07	86057,54
								Mean	86057,81
								Clock	— 0,20
								Temp.	+ 0,09
	71,6	mean							86057,70

Slider 23 divisions. Clock gaining 0″,30 on mean time.			Great weight *above.*					Barometer 29,95.	
	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
	°	m. s.	°	°				s	
June 23rd.	73,0	9. 8	1,22						
		17.28	1,02	1,12	500	498	86054,40	2,05	86056,45
		25.49	0,87	0,94	501	499	86055,09	1,44	86056,53
		34.10	0,74	0,80	501	499	86055,09	1,04	86056,13
	73,1	42.31	0,63	0,68	501	499	86055,09	0,75	86055,84
								Mean	86056,24
								Clock	+ 0,30
	73,1	mean							86056,54
D.				Great weight *below.*					
	72,4	27.33	1,21						
		35 54	1,09	1,15	501	499	86055,09	2,16	86057,25
		44.15	0,99	1,04	501	499	86055,09	1,76	86056,85
		52.36	0,89	0,94	501	499	86055,09	1,44	86056,53
	72,8	0.58	0,79	0,84	502	500	86055,78	1,15	86056,93
								Mean	86056,89
								Clock	+ 0,30
								Temp.	— 0,22
	72,6	mean.							86056,97

The pendulum was now taken down to re-measure the distance between the knife edges, in order to ascertain whether or not they had suffered from use.

The pieces A, a, B and b, being applied as before, the following measurements were taken.

Distance from A to a, 329,06 divisions. B to b, 366,97.				
Date.	Readings of the micrometer.			Divisions. + 39,4 in.
	A to a.	B to b.	Scale.	
June 25th.	9,7 7,0 10,0	39,0 37,3 36,5	630,0 630,0 630,7	953,66 955,86 955,46
The pieces made to change places				
26th.	59,0 59,0 51,0 43,0 41,5	87,0 84,0 75,0 67,7 68,0	680,0 680,3 671,0 664,5 662,5	955,01 956,81 955,51 957,16 955,76
Mean of the whole				955,65

Hence the distance between the knife edges is 39,4 inches +955,65 divisions of the micrometer.

Having thus satisfied myself that no injury to the knife edges was to be apprehended from moderate use, the pendulum was again suspended, but now, to my surprise, I found the number of vibrations different from what they were before the re-measurement. This difference became still greater on the following day, and it at length occurred to me that the moisture of the atmosphere must have undergone some change, and that an alteration had been thus occasioned in the weight of the wooden extremities of the pendulum. On referring to the register of the hygrometer

kept by Mr. BROWNE, it was found that a considerable change had in fact suddenly taken place from moisture to dryness; and, so great was the derangement of the pendulum from this apparently trivial cause, that it became necessary to move the second weight. This was accordingly done, and the following experiments made for again bringing the number of vibrations to an equality.

Slider 29 divisions. Clock gaining 0",18 on mean time.

Great weight *above*. Second weight moved.

Barometer 29,70.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
July 1st.	°	m. s.	°	°				s.	
	69,1	10.31	1,21						
		18.52	1,01	1,11	501	499	86055,09	2,01	86057,10
	69,1	27.14	0,83	0,92	502	500	86055,78	1,38	86057,16
								Mean	86057,13
								Clock	+ 0,18
	69,1	mean.							86057,31
		Great weight *below*.							
	68,8	47.22	1,23						
		55.44	1,09	1,16	502	500	86055,78	2,20	86057,98
	68,9	4. 6	0,99	1,04	502	500	86055,78	1,76	86057,54
								Mean	86057,76
								Clock	+ 0,18
								Temp.	— 0,09
	68,9	mean							86057,85

The second weight was now securely fixed.

Slider 19 divisions. Clock gaining 0″,18 on mean time.			Great weight *above*.					Barometer 29,70.	
	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
	°	m. s.	°	°				s.	
July 1st.	69,3	0.52	1,21						
		9.17	1,00	1,10	505	503	86057,82	1,98	86059,80
	69,2	17.42	0,86	0,93	505	503	86057,82	1,42	86059,24
								Mean	86059,52
								Clock	+ 0,18
	69,3	mean							86059,70
			Great weight *below*.						
	69,3	22.57	1,25						
		31.21	1,13	1,19	504	502	86057,16	2,31	86059,47
	69,3	39.45	1,00	1,06	504	502	86057,16	1,83	86058,99
								Mean	86059,23
								Clock	+ 0,18
	69,3	mean							86059,41

Slider 21 divisions.
Clock gaining 0″,18
on mean time.

Great weight *above*.

Barometer
29,70.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours	Corr. for arc.	Vibrations in 24 hours.
	°	m. s.	°	°				°	
July 1st.	69,3	28. 9	1,23						
		36.32	1,03	1,13	503	501	86056,47	2,08	86058,55
		44.56	0,89	0,96	504	502	86057,16	1,51	86058,67
	69,3	53.21	0,74	0,81	505	503	86057,82	1,07	86058,89
								Mean	86058,70
								Clock	+ 0,18
	69,3	mean							86058,88

E. Great weight *below*.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours	Corr. for arc.	Vibrations in 24 hours.
	69,3	49. 6	1,21						
		57.29	1,09	1,15	503	501	86056,47	2,16	86058,63
		5.53	0,98	1,03	504	502	86057,16	1,73	86058,89
		14.17	0,88	0,93	504	502	86057,16	1,41	86058,57
	69,3	22.42	0,78	0,83	505	503	86057,82	1,13	86058,95
								Mean	86058,76
								Clock	+ 0,18
	69,3	mean.							86058,94

Slider 20 divisions. Clock gaining 0″,18 on mean time.

Great weight *above*.

Barometer 29,70.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
	°	m. s.	°	°				s.	
July 1st.	69,3	1.46	1,34	1,23	502	500	86055,78	2,47	86058,25
		10. 8	1,12	1,03	503	501	86056,47	1,73	86058,20
		18.31	0,94	0,86	505	503	86057,82	1,21	86059,03
		26.56	0,79	0,73	506	504	86058,49	0,87	86059,36
	69,3	35.22	0,68						
								Mean	86058,71
								Clock	+ 0,18
	69,3	mean							86058,89

F.

Great weight *below*.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
F.	69,3	40.28	1,23	1,17	503	501	86056,47	2,23	86058,70
		48.51	1,11	1,04	503	501	86056,47	1,77	86058,24
		57.14	0,98	0,93	504	502	86057,16	1,41	86058,57
		5.38	0,88	0,84	506	504	86058,49	1,15	86059,64
	69,4	14. 4	0,80						
								Mean	86058,79
								Clock	+ 0,18
								Temp.	+ 0,04
	69,4								86059,01

Slider 20 divisions.
Clock gaining 0″,18 on mean time.

Great weight *above*.

Barometer. 29,70.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
July 2nd.	° 68,5	m. s. 33. 7	° 1,41	°				s.	
		41.29	1,18	1,29	502	500	86055,78	2,72	86058,50
		49.54	0,97	1,07	505	503	86057,82	1,87	86059,69
		58.18	0,82	0,89	504	502	86057,16	1,29	86058,45
	68,5	6.43	0,71	0,76	505	503	86057,82	0,94	86058,76
								Mean	86058,85
								Clock	+ 0,18
	68,5	mean							86059,03
G.		**Great weight *below*.**							
	68,0	51.20	1,17						
		59.44	1,04	1,10	504	502	86057,16	1,98	86059,14
		8.10	0,94	0,94	506	504	86058,49	1,44	86059,93
		16.34	0,84	0,89	504	502	86057,16	1,29	86058,45
	68,0	25. 0	0,75	0,79	506	504	86058,49	1,02	86059,51
								Mean	86059,26
								Clock	+ 0,18
								Temp.	— 0,22
	68,0	mean							86059,22

Slider 18 divisions.
Clock gaining 0″,18 on mean time.

Great weight *above*.

Barometer 29,70.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
	°	m. s.	°	°				s.	
July 2nd.	68,6	18.31	1,25						
		26.55	1,04	1,14	504	502	86057,16	2,12	86059,28
		35.19	0,89	0,96	504	502	86057,16	1,50	86058,66
		43.46	0,74	0,81	507	505	86059,16	1,07	86060,23
	68,7	52.11	0,63	0,68	505	503	86057,82	0,75	86058,57
								Mean	86059,18
								Clock	+ 0,18
	68,7	mean							86059,36
H.		Great weight *below*.							
	68,8	57.41	1,23						
		6. 4	1,10	1,16	503	501	86056,47	2,20	86058,67
		14.28	1,00	1,05	504	502	86057,16	1,80	86058,96
		23.52	0,88	0,94	504	502	86057,16	1,44	86058,60
	69,1	31.18	0,82	0,85	506	504	86058,49	1,18	86059,67
								Mean	86058,98
								Clock	+ 0,18
								Temp.	+ 0,09
	68,9	mean							86059,25

Slider 18 divisions. Clock gaining 0",18 on mean time.			Great weight *above.*					Barometer. 29,70.	
	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
	°	m. s.	°	°				s.	
July 2nd.	69,4	17.43	1,24						
		26. 7	1,03	1,13	504	502	86057,16	2,08	86059,24
		34.31	0,88	0,95	504	502	86057,16	1,47	86058,63
		42.55	0,74	0,81	504	502	86057,16	1,07	86058,23
	69,3	51.22	0,64	0,69	507	505	86059,16	0,78	86059,94
								Mean	86059,01
								Clock	+ 0,18
	69,3	mean							86059,19
I.				Great weight *below.*					
	69,3	38.22	1,20						
		46.44	1,08	1,14	502	500	86055,78	2,12	86057,90
		55. 9	c,98	1,03	505	503	86057,82	1,73	86059,55
		3.33	0,88	0,93	504	502	86057,16	1,42	86058,58
	69,3	11.58	0,80	0,84	505	503	86057,82	1,15	86058,97
								Mean	86058,75
								Clock	+ 0,18
	69,3	mean							86058,93

Slider 18 divisions.
Clock gaining 0″,18
on mean time.

Great weight *above*.

Barometer
29,70.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
	°	m. s.	°	°				s	
July 2nd.	69,3	53.29	1,31	1,19	503	501	86056,47	2,31	86058,78
		1.52	1,08	1,00	504	502	86057,16	1,63	86058,79
		10.16	0,92	0,84	506	504	86058,49	1,15	86059,64
		18.42	0,76	0,71	505	503	86057,82	0,82	86058,64
	69,3	26. 7	0,66						
								Mean	86058,96
								Clock	+ 0,18
	69,3	mean							86059,14
K.	Great weight *below*.								
	69,3	32.18	1,24	1,17	502	500	86055,78	2,23	86058,01
		40.40	1,10	1,05	504	502	86057,16	1,80	86058,96
		49. 4	1,00	0,95	505	503	86057,82	1,47	86059,29
		57.29	0,90	0,85	504	502	86057,16	1,18	86058,34
	69,3	5.53	0,81						
								Mean	86058,65
								Clock	+ 0,18
	69,3	mean							86058,83

Slider 19 divisions. Clock gaining 0″.18 on mean time.		Great weight *above*.							Barometer 29,90.
	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
July 3rd.	° 68,0	m. s. 2.18	° 1,28					s.	
		10.42	1,06	° 1,17	504	502	86057,16	2,23	86059,39
		19. 6	0,91	0,98	504	502	86057,16	1,57	86058,73
		27.31	0,74	0,82	505	503	86057,82	1,10	86058,92
	68,2	35.57	0,64	0,69	506	504	86058,49	0,77	86059,26
								Mean	86059,08
								Clock	+ 0,18
	68,1	mean							86059,26
L.		Great weight *below*.							
	67,7	23.52	1,29						
		33.16	1,14	1,21	504	502	86057,16	2,39	86059,55
		40.40	1,01	1,07	504	502	86057,16	1,87	86059,03
		49. 4	0,93	0,97	504	502	86057,16	1,54	86058,70
	67,8	57.30	0,72	0,82	506	504	86058,49	1,10	86059,59
								Mean	86059,22
								Clock	+ 0,18
								Temp.	− 0,18
	67,8	mean							86059,22

Slider 19 divisions.
Clock gaining 0″,18 on mean time.

Great weight *above.*

Barometer 29,90.

	Temp.	Time of coincidence.	Arc of vibration	Mean arc.	Interv. in seconds.	No. of vibrats.	Vibrations in 24 hours.	Corr. for arc.	Vibrations in 24 hours.
	°	m. s.	°	°				s	
July 3rd.	68,3	45. 3	1,23	1,13	504	502	86057,16	2,08	86059,24
		53.27	1,03	0,95	504	502	86057,16	1,47	86058,63
		1.51	0,87	0,80	505	503	86057,82	1,04	86058,86
		10.16	0,74	0,68	506	504	86058,49	0,75	86059,24
	68,4	18.42	0,63						
								Mean	86058,99
								Clock	+ 0,18
	68,4	mean							86059,17
M.		**Great weight *below.***							
	68,4	24.31	1,24	1,17	503	501	86056,47	2,23	86058,70
		32.54	1,11	1,05	504	502	86057,16	1,80	86058,96
		41.18	0,99	0,94	504	502	86057,16	1,44	86058,60
		49.42	0,90	0,86	506	504	86058,49	1,20	86059,69
	68,5	58. 8	0,82						
								Mean	86058,99
								Clock	+ 0,18
								Temp.	+ 0,04
	68,5	mean							86059,21

The results of such of the preceding experiments as are to be used for calculating the length of the seconds pendulum, are brought under one view in the following table:

Place of the slider	Expt.	Temp.	Barom.	No. of vibrations. Great wt. *above*.	Diff.	No. of vibrations. Great wt. *below*.	Vibs. in excess or defect.
23	A	68,7	29,76	86059,39	,03	86059,42	—
23	B	71,3	29,86	86057,70	,23	86057,93	—
23	C	71,4	29,86	86057,93	,23	86057,70	+
23	D	73,1	29,95	86056,54	,43	86056,97	—
Pendulum re-measured.							
21	E	69,3	29,70	86058,88	,06	86058,94	—
20	F	69,3	29,70	86058,89	,12	86059,01	—
20	G	68,5	29,70	86059,03	,19	86059,22	—
18	H	68,7	29,70	86059,36	,11	86059,25	+
18	I	69,3	29,70	86059,19	,16	86058,93	+
18	K	69,3	29,70	86059,14	,31	86058,83	+
19	L	68,1	29,90	86059,26	,04	86059,22	+
19	M	68,4	29,90	86059,17	,04	86059,21	—
			Mean	86058,71		86058,72	

No other explanation of this table appears to be necessary, than that the column entitled "Difference" expresses the difference between the number of vibrations in the two positions of the pendulum, and that the last column indicates by the sign + or — whether the number of vibrations exceeds or falls short of the truth; which inference is drawn from a comparison of the number of vibrations when the great weight is above, with the number in that position of the pendulum when the great weight is below. The mean of the vibrations in the column "Great weight *above*" not differing sensibly from that headed "Great weight *below*," is a proof that the number of vibrations in either position of the pendulum may be considered as equal, and consequently that the

one knife edge being the point of suspension, the other must necessarily be in the centre of oscillation.

Length of the pendulum vibrating seconds.

The distance between the knife edges was as follows:

	Inches.		Divisions.		Inches.
By the 1st measurement	39,4	+	956,47	=	39,44094
By the 2nd, -	39,4	+	954,49	=	39,44086
By the 3d, -	39,4	+	955,65	=	39,44090
				Mean	39,44090
Corr. for error in division of the scale (see page 54) - -					— 0,00005
					39,44085

Hence, 39,44085 inches may be taken as the distance between the knife edges at the temperature of 62 degrees.

Using the vibrations when the great weight was *below*, as being nearer to the truth than in the other position of the pendulum, we obtain the following results.

Expt.	Temp.	Barom.	Vibrations in 24 hours.	Length of the seconds pen. in air	Corr. for the atmosphere.	Length of the scs. pend. in vacuo.	Difference from the mean.
A	68,7	29,76	86059,42	39,13313	,00544	39,13857	+,00028
B	71,3	29,86	86059,93	39,13278	,00544	39,13822	—,00007
C	71,4	29,86	86057,70	39,13260	,00544	39,13804	—,00025
D	73,1	29,95	86056,97	39,13259	,00544	39,13803	—,00026
E	69,3	29,70	86058,94	39,13293	,00544	39,13837	+,00008
F	69,3	29,70	86059,01	39,13298	,00544	39,13842	+,00013
G	68,5	29,70	86059,22	39,13286	,00545	39,13831	+,00002
H	68,7	29,70	86059,25	39,13296	,00544	39,13840	+,00011
I	69,3	29,70	86058,93	39,13291	,00544	39,13834	+,00005
K	69,3	29,70	86058,83	39,13282	,00544	39,13825	—,00003
L	68,1	29,90	86059,22	39,13271	,00548	39,13819	—,00009
M	68,4	29,90	86059,21	39,13281	,00548	39,13829	—,00000
					Mean	39,13829	

The length of the pendulum thus obtained requires yet another correction to reduce it to what it would have been at the level of the sea. The elevation of the apartments of the Royal Society at Somerset House above low-water mark, is known to be 81 feet; and by several careful observations with an excellent mountain barometer by RAMSDEN, I found the room in Portland Place, in which the experiments were made, to be two feet below the Royal Society's apartments; and as the height of the pendulum above the floor was four feet, we obtain 83 feet for the elevation of the pendulum above the level of the sea. Now the force of gravity increasing inversely as the square of the distance from the earth's centre, the length of the pendulum must be increased in the same proportion, and taking the radius of the earth for the latitude of Portland Place to be 3954,583 miles, we have 39,1386 inches for the length of the pendulum vibrating seconds at the level of the sea.

It may be remarked that the greatest difference between the mean result and that of any one of the twelve sets of experiments contained in the preceding table, is only ,00028 of an inch, or $\frac{1}{134959}$ of the whole length of the pendulum; and as seven of the twelve sets are within one ten thousandth of an inch of the mean result, it may be inferred that the above determination cannot be very distant from the truth.

The length here given, is that required to perform one vibration in $\frac{1}{86400}$ part of a mean *solar* day, this being the measure of time usually employed for the purpose; but I am at a loss to conjecture why this is preferred to the sidereal day, a measure of time which marks a complete revolution

of the earth, and is readily obtained, being the interval between the returns of any fixed star to the meridian.

I shall now proceed to notice the sources of error which may be supposed to have affected the results of the preceding experiments.

These may be classed under the following heads.

1. The measurement of the distance of the knife edges.
2. The number of vibrations in 24 hours.
3. The temperature, and
4. The form of the knife edges.

On the first, it is scarcely necessary to offer any remark. Since the mean results of three several sets of measurements are within one ten-thousandth of an inch of each other, and the different methods employed, preclude, it may be presumed, any accidental coincidence, we may with confidence infer that the error in the distance of the knife edges, cannot amount to one ten-thousandth of an inch.

Among the number of vibrations in 24 hours given in the various sets of experiments, there appear to be differences which amount in some instances to 1,6. These differences however do not influence the truth of the result, beyond a certain minute quantity, the extent and origin of which I shall proceed to explain.

In order to determine the vibrations in 24 hours, it is necessary to ascertain the number of vibrations and parts of a vibration made by the brass pendulum during a certain number of *complete* seconds; but the moment of observation being limited to that when the brass pendulum is at the lowest part of the arc, the process is of necessity reversed, and the brass pendulum is observed to make a certain number of *complete* vibrations,

during a certain number of seconds and parts of a second which constitute *the interval.* The disappearance of the disk can however be noted only to a single second, and the brass pendulum may arrive at the lowest part of the arc either precisely at this second, or at any portion of a second preceding it. An error might possibly arise from this circumstance amounting to nine tenths of a second, by which the interval deduced from observation would be less than the truth, and as an error of one second in the interval, occasions a difference of 0,63 in the number of vibrations in 24 hours, if 0,55 (the proportional part of 0,63) be divided by 4 (the number of intervals forming each set of experiments) we have 0,14 for the greatest error *in defect* in the number of vibrations in 24 hours which can arise from this cause.

On the contrary, if the *second* coincidence or return of the brass pendulum to the lowest point of the arc, should have taken place nine tenths of a second before the second at which the disappearance of the disk was noted, the error in the number of vibrations in 24 hours would amount to the same quantity, and would now be *in excess.*

If the first and third coincidences take place accurately at the time of the observed disappearance of the disk, and the observation of the second coincidence should differ nine tenths of a second from the truth, it is obvious that the number of vibrations in 24 hours deduced from each interval will be erroneous about 0,56 the one being *in excess*, the other *in defect*: The mean of both will be the truth, though the observed difference between the two amounts to so considerable a quantity as 1,2.

The last coincidence of each set, takes place when the arc

of vibration is much reduced. It is therefore not impossible that an error of one second may sometimes, though rarely, occur in determining the time of this coincidence. This would occasion an error of about 0,63 in the number of vibrations in 24 hours, which divided by 4 as before, would influence the mean result 0,15 of a vibration.

In estimating these errors, I have taken an extreme case, as it is probable they would in most instances be compensated by the succeeding intervals. Supposing them however to be combined, the greatest effect on the mean result of any one set of experiments might amount to about 0,3 of a vibration in 24 hours, and the difference between the number of vibrations in either position of the pendulum, might have been double this quantity, and yet when the great weight was below, not have differed from the truth more than 0,3 of a vibration.

It appears then, that if the experiments have been conducted with sufficient care, no greater difference should be found between the mean, and any one of the resulting lengths of the pendulum contained in the preceding table, than might have been occasioned by a difference of 0,30 f a vibration in 24 hours, and this is found to be about 0,0003 of an inch.

In fact, on referring to the table we perceive that the experiments A and D, which differ most from the mean, give, the one, 00029 of an inch *in excess*, and the other ,00026 *in defect*.

In considering the sources of error, it may not be unnecessary to remark that had the bar of the pendulum been made too thick, and the knife edges not been placed accurately at right angles to it, an error, though very minute, might have arisen from the effect of the obliquity in diminishing the dis-

tance of the centre of oscillation from the axis. This was sufficiently guarded against by having the bar so thin as to ensure its becoming perpendicular by its own weight, had the position of the knife edge been in a small degree erroneous; for though the form the bar would assume is strictly speaking a curve, it may without sensible error be considered as a straight line.

With regard to temperature, every precaution was taken to prevent error. The thermometer used was made by Mr. Troughton for the late Sir George Shuckburgh. It is divided into half degrees, and the height of the mercury may be estimated to one tenth of a degree. It has been already observed in the preceding part of this paper, that the thermometer was approached only at the first and last coincidences.

The experiments themselves afford, it is presumed, a sufficient proof of the stability of the knife edges. Every care was taken to form them in the first instance as perfect as possible; and after four sets of experiments had been made, they were found on re-measurement to have suffered no perceptible alteration; and it is evident by the near agreement of the results, that they remained uninjured during the succeeding experiments: it is difficult therefore to conceive that any error can have arisen from this source.

I may here remark, that the method I have employed in determining the length of the pendulum, possesses other advantages besides that of superseding the errors arising from unequal density or figure; and one, not the least considerable is, that after a very few vibrations, the true length of the pendulum is bounded by certain known limits. Thus in the two first sets of experiments, after the re-measurement of the

distance between the knife edges, we may remark that when the slider was at 29 divisions, the number of vibrations (the great weight being below) was, 86057,85 and in *defect;* and when the slider was removed to 19 divisions, the number of vibrations was 86059,41 and in *excess*. The true number of vibrations then is evidently between the two, and the utmost extent of error in using either of these numbers must fall short of 1,73 their difference when reduced to the same temperature. But if the mean be employed in the computation, the length of the pendulum will be found to differ only about four ten-thousandths of an inch from the mean result given in the foregoing table.

It may not be unnecessary to add that every experiment made has been retained; nor do I consider any one as less entitled to credit than the rest, excepting that marked A, in the table; and that, only because the rate of the clock was not observed on the day of the experiment, but was taken to be the same as the rate of the following day.

The length then of the pendulum vibrating seconds in vacuo at the level of the sea, measured at the temperature of 62° of FAHRENHEIT, appears to be

		inches.
By Sir G. SHUCKBURGH's standard	-	39,13860
By General ROY's scale	- -	39,13717
By BIRD's Parliamentary standard	-	39,13842

the latitude of the place of observation being 51° 31′ 8″,4 north.*

* The latitude was deduced from the data contained in the trigonometrical survey; Mr. BROWNE's house bearing from Portland Chapel 74°. 38′. 50″. west from the north, the distance being 283 feet. This differs only 0″,1 from the latitude determined by Mr. BROWNE from a great number of observations.

An objection might be urged against the use of the knife edge, on the ground that being an elastic substance it may possibly suffer temporary compression, and thus perhaps introduce a source of error. In order to meet any doubt that might arise on this important part of the subject, it is my intention to commence a series of experiments with a pendulum of the same construction as that which has been described, but vibrating on cylinders instead of knife edges, and I trust soon to have the honor of laying the result before the Royal Society.

London, July, 1817.

APPENDIX.

Since the preceding Paper was written, a very curious and important theorem has been discovered by M. LAPLACE, of which Dr. YOUNG has favoured me with a concise demonstration, together with some other investigations which I shall subjoin in his own words.

MY DEAR SIR,

I cannot forbear to congratulate you on the discovery of the singular property of your pendulum, which has lately been demonstrated by M. LAPLACE, since it appears to remove the only doubt, that could reasonably be entertained, of the extreme accuracy of the results of your experiments. The correction for the curvature of the rolling surfaces, in the case of a simple pendulum, is very easily obtained from the geometrical determination of the curve described, although M. LAPLACE's train of reasoning, from mechanical principles, is somewhat too elaborate to be readily followed through all the symbols in which it is enveloped: and the same geometrical considerations appear, at first sight, to be equally applicable to the case of compound pendulums in general, since the motions of all their effective parts are concentric with those of a simple one similarly suspended. But upon further reflection, it becomes evident that these motions, though concentric, are related to each other in proportions somewhat different from those of a similar pendulum vibrating on a single point, and it is therefore necessary to deter-

mine the modification of the motion produced by this difference of connexion. The investigation may however be conducted in a method much more simple and intelligible to ordinary capacities, than that which has been adopted by the celebrated mathematician to whom we are indebted for the theorem; and I am tempted to send you an "*aperçu*" of the reasoning by which I have satisfied myself respecting it.

It follows immediately from the general theorem for finding the curvature of trochoids of all kinds, (Lectures on Nat. Phil. II. p. 559) that the radius of curvature of the path of any point, in the rod of a pendulum supported by a cylindrical axis, will initially be a third proportional to the distances of the point from the centre of the cylinder, and from the surface on which it rolls: so that when the cylinder is small, and the pendulum simple, the centre of curvature of its path may be considered as situated at the distance of the radius r below the point of contact: and this is obviously the only correction required for such a pendulum as that of BORDA. But when the weight is divided, or of considerable magnitude, it becomes necessary to calculate the effect of the different curvatures of the paths of its different parts, and to compare these paths with that of a pendulum A of any given length a. Supposing, for the sake of simplicity, the weight of each horizontal section to be concentrated in the vertical line, and calling the distance of any particle P below the surface of the cylinder x, the radius of curvature of its path will be a third proportional to $x + r$ and x, that is, $\frac{xx}{x+r}$; and the inclination of the curve at a given distance from the vertical line being always directly as the curvature, or inversely as its radius, the force derived from the weight of P will be

to the force at an equal distance in the path of A, as a to $\frac{xx}{x+r}$, or as $\frac{a(x+r)}{xx}$ to 1. Now the point of the rolling pendulum confined to the vertical line is not the centre of curvature, but initially the surface of the cylinder: so that this must be considered as the point of intersection with the vertical line, and as the fulcrum of the lever; consequently the distance of P from the vertical line will be, to that of the pendulum A, as x to a, and its immediate force will be $\frac{a(x+r)}{xx} \cdot \frac{x}{a} \cdot P = \frac{x+r}{x}P$; but this force, acting only at the end of a lever x, will have its effect at A again reduced in the ratio of x to a, and will then become $\frac{x+r}{a}$ P: and if we express the sum of all the similar forces belonging to the body by the character Σ, whether found by a fluxional calculation or otherwise, we have the whole force, at A, $\Sigma \frac{x+r}{a}$ P. The reduced or rotatory inertia of the body, sometimes very improperly called the "momentum" of inertia, will also be expressed by $\Sigma \frac{xx}{aa}$ P, being reduced in the ratio of the squares of the distances from the fulcrum; consequently the accelerative force will be to that of the pendulum A as $\dfrac{\Sigma \frac{xx}{aa} P}{\Sigma \frac{x+r}{a} P}$ to 1, or as $\frac{\Sigma\, xx\, P}{a\, \Sigma\, (x+r)\, P}$ to 1; since it is indifferent whether the integral or the differential be divided by the constant quantity a: and in order to express the length of the equivalent pendulum, we must suppose a to be as much lengthened as the force is weakened, so that we have for this length $\frac{\Sigma\, xx\, P}{\Sigma\, (x+r)\, P}$. It is obvious that the denominator of this fraction is the same that would express the force of the body with regard to the centre of the cylinder as a fixed point; and it might indeed

have been inferred at once, from the principle of virtual velocities, that the force must be the same in either case, however irregular the form of the body may be: but it is somewhat more satisfactory to follow the mechanical steps by which the operation of the law takes place. If we make $r = 0$, we have $\frac{\Sigma xx P}{\Sigma x P} = l$, for the length of the equivalent pendulum when the surface of the cylinder is supposed to be the centre of suspension; and it follows from the well known properties of the centre of gravity, that $\Sigma x P$ the sum of the product of all the particles into their distances, is equal to Qd, the product of the whole weight Q into the distance of the centre of gravity from the point of suspension; and $\Sigma x^2 P = \Sigma x P l = d Q l$, so that the equivalent length for the rolling pendulum becomes $\frac{dlQ}{\Sigma (x+r) P} = \frac{dlQ}{\Sigma x P + \Sigma r P} = \frac{dlQ}{dQ + rQ} = \frac{l}{1 + \frac{r}{d}} = l\left(1 - \frac{r}{d}\right)$, r being supposed very small; which, for a simple pendulum, when $d = l$, becomes $l - r$, as it ought to do. We must however find the displacement of the centre of suspension which is capable of producing an equal alteration in the length of the equivalent pendulum; and for this purpose we must have recourse to the theorem of HUYGENS, which may be easily deduced from the expression $\frac{\Sigma xx P}{dQ}$: for calling $x - d$, the distance of any particle of the body from its centre of gravity, y, we have $x^2 = (d + y)^2 = d^2 + 2dy + y^2$, and $\Sigma x^2 P = \Sigma d^2 P + 2d\Sigma y P + \Sigma y^2 P = d^2 Q + 0 + \Sigma y^2 P$, the integral of $\Sigma y P$, the product of the distance of each particle into its distance from the common centre of gravity always vanishing: consequently $l = \frac{\Sigma yy P + d^2 Q}{dQ} = \frac{\Sigma yy P}{dQ} + d$, and $l - d = \frac{\Sigma yy P}{dQ}$; which is HUYGENS's theorem: the constant quantity $\frac{\Sigma yy P}{Q}$ being equal

to $dl - d^2$. If now we suppose d to be increased by the small quantity s, the reciprocal, instead of $l - d$, will become $\frac{dl - dd}{d + s} = \frac{l - d}{1 + \frac{s}{d}} = (l - d)\left(1 - \frac{s}{d}\right) = l - d - l\frac{s}{d} + s$, to which adding $d + s$, we have $l - l\frac{s}{d} + 2s$, the increase of the length being $\frac{2d - l}{d}s$; and making this equal to $-\frac{l}{d}r$, we have $s = \frac{-lr}{2d - l}$: and when the pendulum is inverted, substituting $l - d$ for d, the expression becomes $\frac{-lr}{2l - 2d - l} = \frac{lr}{2d - l}$, which, added to the former negative value of the same quantity, must always destroy it: so that the length of the equivalent pendulum will be truly measured by the simple distance of the surfaces of the cylinders, as M. LAPLACE has demonstrated.

There is however another correction, of which it becomes necessary to determine the value, when a very sharp edge is used for the axis of motion, as in the pendulum which you have employed: since it appears very possible, that in this case the temporary compression of the edge may produce a sensible elongation of the pendulum. But it will be found, by calculating the magnitude of this change, that when the edge is not extremely short, and when its bearing is perfectly equable, this correction may be safely neglected.

Supposing a to be the distance from the edge, in the plane bisecting its angle, at which the thickness is such, that the weight of the modulus of elasticity corresponding to the section shall become equal to the weight of the pendulum, the elasticity at any other distance x from the edge will be measured by x, while the weight is represented by a; so that the elementary increment x' will be reduced by the pressure of the weight to $\frac{x}{a + x}x'$, and the element of the compression

will be $\frac{a}{a+x}x'$, and its fluxion $\frac{a}{a+x}dx$, of which the fluent is $a\,\mathrm{HL}\frac{a+x}{a}$. Now the height of the modulus of elasticity of steel is ten million feet, (Lect. Nat. Phil. II. p. 509) and the weight of a bar, an inch square, and of this height, would be about 30 millions of pounds; so that if the weight be 10 pounds, and the line of bearing an inch long, the thickness at the distance a must be one three millionth of an inch; and supposing the angle a right one, a must be $\frac{1}{4244000}$; and making $x = 1$, we have the whole compression of the edge within the depth of an inch $\frac{1}{4244000}$ HL 4244001; and this logarithm being 15.26, the correction becomes equal to the 360 thousandth of an inch. If the bearing were one tenth of an inch only, the compression for both the opposite edges would become $\frac{1}{18000}$, supposing that they retained their elasticity, and underwent no permanent alteration of form. In fact, however, the edge must be considered as a portion of a minute cylinder, which will be still less compressible than an angle contained by planes; and the happy property, demonstrated by M. LAPLACE, will prevent any sensible inaccuracy from this cause, however blunt the edges may be, supposing that the steel is of uniform hardness in both.

Believe me, my dear Sir, very sincerely yours,

THOMAS YOUNG.

Welbeck Street, 5th Jan. 1818.

P. S. It is easy to show that the determination of the length of the pendulum, by means of a weight sliding on a rod or bar, which is the method that I have proposed as the most convenient for obtaining a correct standard, is equally independent of the magnitude of the cylinder employed. The

reduced inertia $\Sigma x^2 P$ here consists of two portions: for the rod we may take the equivalent expression dlQ, which we may call axy, a being the weight of the bar (Q), x the distance (d) of the centre of gravity, and y the equivalent length (l): for the ball we must employ the formula $\Sigma x^2 P = \Sigma y^2 P + d^2 Q$, and call $\Sigma y^2 P$, u, and $d^2 Q$, bz^2, b being the weight of the ball, and z the distance of its centre of gravity from the point of suspension: and in the same manner the force $\Sigma(x + r)P = (d+r)\,Q$ must be composed of the two portions $a\,(x+r)$ and $b\,(z+r)$, so that the equivalent length becomes $\frac{axy + u + bzz}{a\,(x+r) + b\,(z+r)} = \frac{z^2 + \frac{axy + u}{b}}{z + \frac{ax + ar + br}{b}}$; which we may call $\frac{zz + v}{z + w} = t$. The experiment being then performed in four different positions of the weight, at the distances d', d'', and d''', so that the second value of z may be $z - d' = z'$, the third $z - d'' = z''$, and the fourth $z - d''' = z'''$, we must observe the times of vibration, and deduce from them the comparative lengths of the equivalent pendulum, t, $n't$, $n''t$, and $n'''t$: and hence the value of z, of v, and of t may be obtained, without determining w, and of course without employing the quantity r.

First, $\frac{z^2+v}{z+w} = t$, $\frac{z'^2+v}{z'+w} = n't$, $\frac{z''^2+v}{z''+w} = n''t$, $\frac{z'''^2+v}{z'''+w} = n'''t$.

II, $z + w = \frac{z^2+v}{t}$, $z' + w = \frac{z'^2+v}{n't}$, $z'' + w = \frac{z''^2+v}{n''t}$, $z''' + w = \frac{z'''^2+v}{n'''t}$.

III, $z - z' = d'$; $z - z'' = d''$; $z - z''' = d'''$.

IV, $d' = \frac{z^2+v}{t} - \frac{z'^2+v}{n't}$; $d'' = \frac{z^2+v}{t} - \frac{z''^2+v}{n''t}$; $d''' = \frac{z^2+v}{t} - \frac{z'''^2+v}{n'''t}$.

V, $t = \frac{z^2+v}{d'} - \frac{z'^2+v}{n'd'} = \frac{z^2+v}{d''} - \frac{z''^2+v}{n''d''} = \frac{z^2+v}{d'''} - \frac{z'''^2+v}{n'''d'''}$.

VI, by comparing the first of these equations successively with the second and third, and bringing the terms containing v to the same side, we obtain

$$v = \left(\frac{z^2}{d'} - \frac{z'^2}{n'd} - \frac{z^2}{d''} + \frac{z''^2}{n''d''}\right) : \left(\frac{1}{d''} - \frac{1}{n''d''} - \frac{1}{d'} + \frac{1}{n'd'}\right) = \left(\frac{z^2}{d'} - \frac{z'^2}{n'd} - \frac{z^2}{d'''} + \frac{z'''^2}{n'''d'''}\right) : \left(\frac{1}{d'''} - \frac{1}{n'''d'''} - \frac{1}{d'} + \frac{1}{n'd'}\right).$$

This equation contains only the squares of the values of z with known coefficients; and if we substitute $z-d'$, $z-d''$, and $z-d'''$ for z', z'', and z''', respectively, we shall obtain an equation in the form $ez^2+fz=g$, whence $z = \pm\sqrt{(g+\frac{1}{4}f^2)} - \frac{1}{2}f$.

T. Y.

Fig. 1.

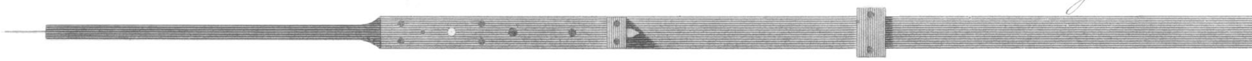


Fig. 2.

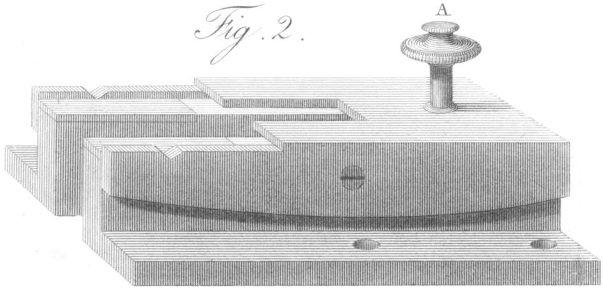
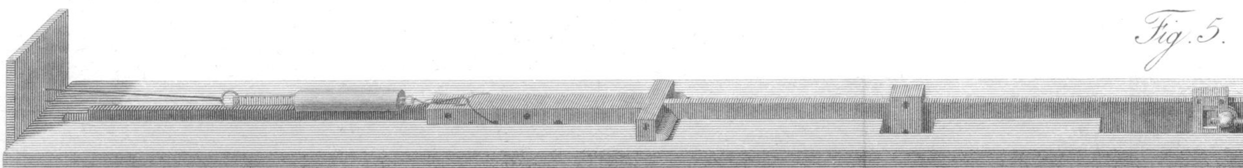


Fig. 5.



Mary F. Kater, del.

Fig. 1.

2.

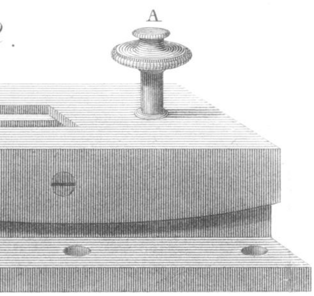


Fig. 3.

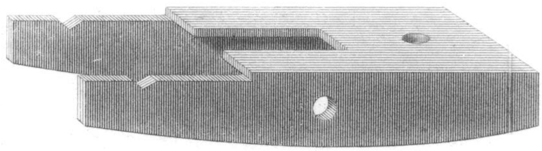
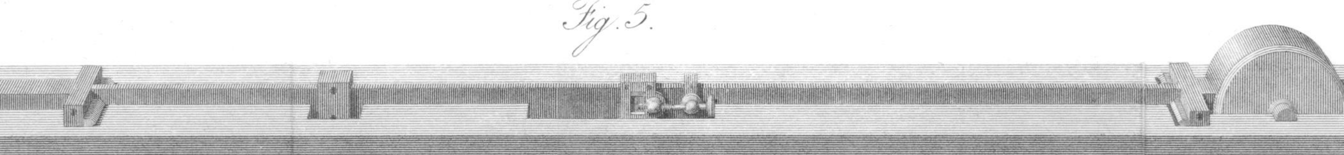


Fig. 5.



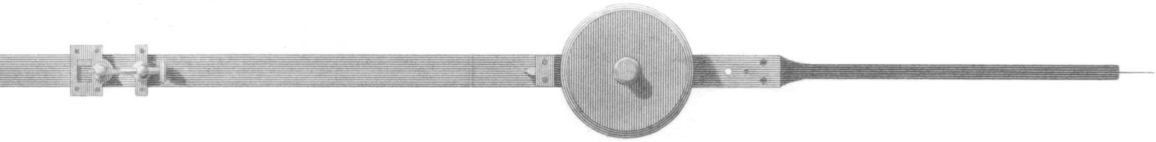


Fig. 3.

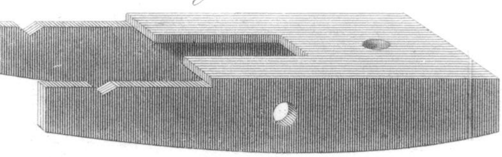
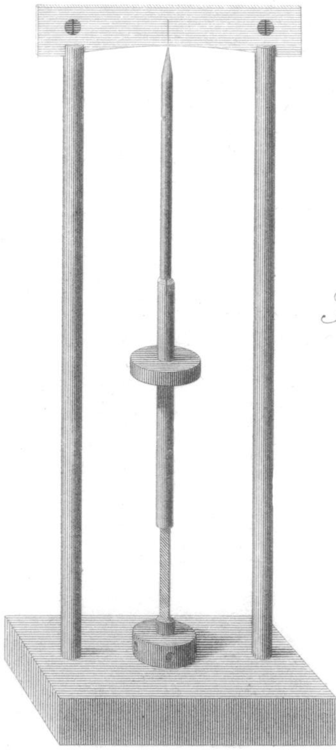
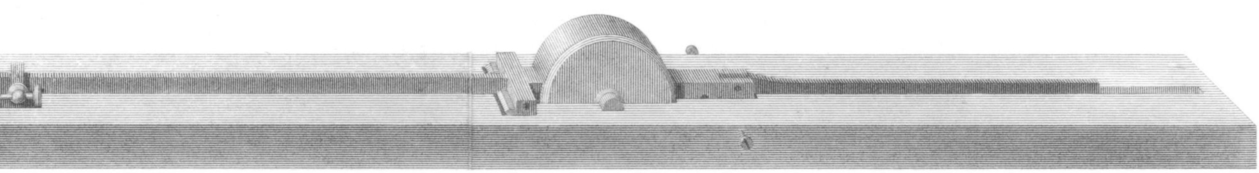
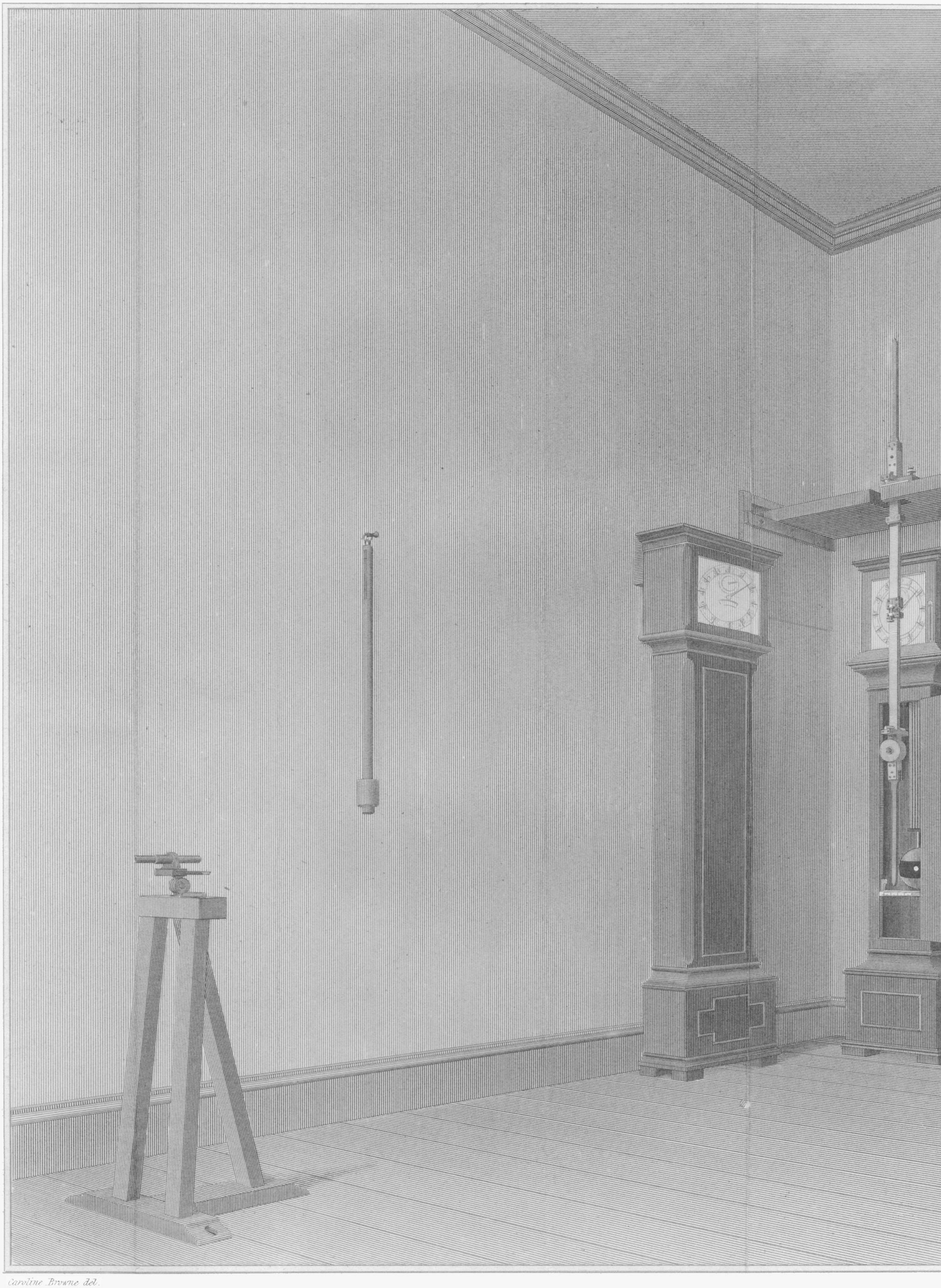


Fig. 4.





W. Lowry, Sculp.



Caroline Browne del.

Engraved by W. Lowry.